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Turbulent flow characteristics of viscoelastic fluids

By A. B. METZNER AND M. GRAHAM PARK†
University of Delaware, Newark, Delaware

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In this paper the turbulent flow characteristics of viscoelastic fluids are investigated quantitatively. The outstanding property of these flow fields is seen to be a very pronounced suppression of turbulence, accompanied by major reductions in the turbulent drag coefficients. Careful measurements of the rheological properties of the several fluids used suggest that the observed turbulence suppression is a function of the ratio of the elastic to the viscous forces developed in the fluid. An empirical correlation of the results, based upon this observation, is proposed; the present data, while indicative, are not sufficiently extensive to verify conclusively the existence of a quantitative correlation.

In a number of respects, the observed reduction in drag is similar to that which may be obtained through promotion of 'slip' at the tube wall or by addition of particulate matter. It is shown that slip phenomena are clearly distinct from those studied in the present work but that particulate effects (albeit of much greater magnitude than observed heretofore) cannot be ruled out as contributory mechanisms. Further studies are thus required to determine the relative importance of continuum (viscoelastic) and particulate effects.

Introduction, statement of problem

A useful similarity criterion for the study of purely-viscous non-Newtonian fluids has been shown to be the generalized Reynolds number, $N'_{Re} = D^{n'}V^{2-n'}\rho/\gamma$ (Metzner & Reed 1955; Dodge & Metzner 1959). Here, n' denotes the flow behaviour index of the fluid and γ $(= g_c K' 8^{n'-1})$ reflects the fluid consistency through the consistency index K'; D is the tube diameter, V the mean fluid velocity, and ρ the fluid density; g_c is a conversion factor depending on the units employed. The flow behaviour index n' and consistency index K' are related through the relationship

$$\tau_w = K'(8V/D)^{n'}, \tag{1}$$

where τ_w is the wall shear stress, which is valid for all fluids under conditions of steady, well-developed laminar flow through round tubes, depending only upon the usual assumptions of a fluid which behaves as a continuum and exhibits no slip at the tube wall (Metzner 1961). In the special case of materials exhibiting a power-law constitutive relationship, K' and n' become constants related to the power-law coefficients, but in general such behaviour may not be exhibited by real materials nor is it required in order to validate equation (1) or to define the generalized Reynolds number. The flow index n' thus reflects the degree to which

† Present address: E. I. du Pont de Nemours and Company, Wilmington, Delaware.

the rheological properties of a purely viscous fluid, at any chosen value of the wall stress τ_w, diverge from those of Newtonian fluids, i.e. it measures the non-linearity of the fluid behaviour with the special case of a Newtonian fluid corresponding to a flow index of unity. In this event, the fluid consistency K' reduces to the viscosity and the generalized Reynolds number to the classical one for Newtonian fluids.† For most real fluids of interest n' is less than unity.

Expressed in terms of this similarity criterion, studies of turbulence in cylindrical ducted flow fields using polymeric solutions as well as slurries, some of which exhibit non-Newtonian characteristics, have led to the following experimental observations:

(1) The turbulent flow characteristics of *purely viscous* non-Newtonian fluids are generally similar to those of Newtonian fluids. Thus, the turbulent velocity profiles (Bogue & Metzner 1963; Eissenberg & Bogue 1963) are very similar, although perhaps very slightly steeper, than those of Newtonian fluids. The value of the generalized Reynolds number at the transition from laminar to turbulent flow depends slightly upon the detailed rheological properties of the fluid (Dodge & Metzner 1959; Hanks & Christiansen 1962; Hanks 1963), but the maximum variation due to the changes in properties is small.‡ Correspondingly, the turbulent drag coefficients, while clearly dependent on the value of the flow behaviour index of the fluid, usually fall within 50 % of the Newtonian values (see Dodge & Metzner 1959; Thomas 1962; Bogue & Metzner 1963; Eissenberg & Bogue 1963). Furthermore, a straightforward generalization for the usual analyses of turbulent profiles and pressure-loss characteristics for Newtonian fluids (Dodge & Metzner 1959) appears to correlate or interpret the frictional characteristics quite well. These have now been verified in a number of independent studies (Bogue & Metzner 1963; Savins 1963; Park 1963), and, while the accuracy and range of the data are not always as great as desired, the general frictional behaviour appears to be rather well defined at moderate Reynolds numbers. Since the turbulent drag coefficients for purely-viscous non-Newtonian fluids usually fall below those of Newtonian materials, it would appear as if addition of solids or of polymeric materials to a Newtonian solvent or continuous phase, to yield a purely-viscous but non-Newtonian fluid of low flow behaviour index, might serve to reduce pressure drop under turbulent conditions. In fact, however, the 'thickening' action of such additives seems generally to outweigh this reduction in drag coefficient and the net result is almost always a significant increase in the actual pressure drop.

Thus, it is clear from these results that gross changes in the shear-stress–shear-rate behaviour of the fluid, reflected in correspondingly great changes in the laminar velocity profiles, exert only modest influences on the transitional

† The numerical term $8^{n'-1}$ in the denominator of the generalized Reynolds number is introduced so that the drag coefficient–Reynolds number relationships of non-Newtonian materials may superimpose identically upon the usual curve for Newtonian fluids under conditions of well-developed, steady, laminar flow through round tubes.

‡ The correlations proposed by Thomas (1962) are in a different form because of a major difference in the choice of terms in the Reynolds number. When reduced to the basis of the generalized Reynolds number used herein they are not inconsistent with this statement, however.

Reynolds number, upon the turbulent velocity profiles and the turbulent frictional characteristics.

(2) Contrasted to the above behaviour of purely-viscous fluids, the effects noted in *viscoelastic* polymeric solutions (Sailor 1960; Lummus, Fox & Anderson 1961; Crawford 1962, Fabula 1963; Park 1963; Savins 1963) are enormous. The transitional value of the generalized Reynolds number may be increased by as much as an order of magnitude (to beyond 10,000 for tube flow); the turbulent

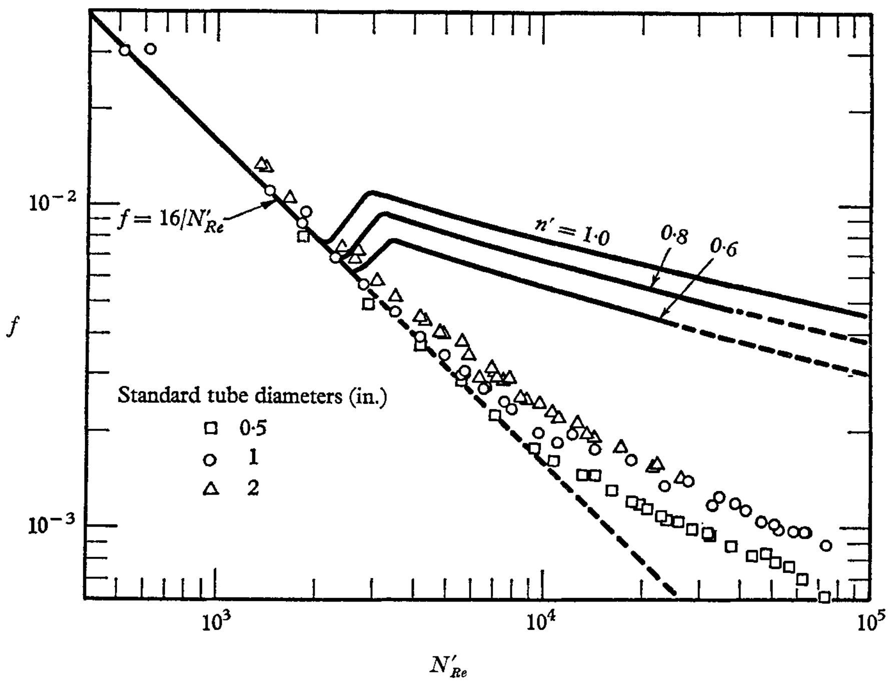

FIGURE 1. Drag coefficient (f) *vs* Reynolds number (N'_{Re}). —: turbulent curves for purely-viscous fluids. △, ○, □: Present study of a 0·3 % solution of J-100 in water (viscoelastic fluid).

drag coefficient may be as much as an order of magnitude lower than those of turbulent Newtonian fluids at the same Reynolds number, and preliminary measurements of the velocity profiles under turbulent-flow conditions (Shaver & Merrill 1959) indicate them to be surprisingly steep. Contrasted also to the effects noted in purely-viscous fluids, the turbulent viscoelastic characteristics may be brought out by addition of very small quantities of the polymer, frequently only a few parts per million of solution.

The striking differences between purely-viscous fluids (including Newtonian fluids) on the one hand and viscoelastic materials on the other, in so far as drag characteristics are concerned, are illustrated in figure 1. The curves shown are those which were previously developed (Dodge 1957, Dodge & Metzner 1959) for purely viscous fluids having flow behaviour indices in the range of interest for the present work. The degree of accuracy with which these turbulent curves are defined may be noted by considering that the standard deviation of the data from the curves was less than 3 %. Implied by these curves is the absence of miscellaneous aberrations: in particular, the drag-coefficient–Reynolds-number–flow-behaviour-index relationships have been shown to be statistically independent of tube diameter over the threefold to fourfold ranges studied. By contrast, the data for the viscoelastic fluid show a clear effect of tube diameter, their transition to turbulence is appreciably delayed and the drag coefficients fall well below (as much as *fivefold below*) those for purely-viscous fluids having similar flow-behaviour indices. Further, the curves appear to be diverging as the Reynolds number increases. This behaviour is similar to that published some years ago (Dodge & Metzner 1959) for less viscoelastic fluids and has since been shown to be exhibited by a large number of fluid systems (Sailor 1960; Crawford 1962; Savins 1963).

Theoretical discussion

The differences between the behaviour of purely-viscous fluids and that of materials such as the J-100 solution depicted in figure 1 could possibly arise for a number of reasons:

(1) There may be particulate (i.e. non-continuum) effects present in the case of the latter materials which promote stability of the laminar flow field or dampen the turbulence, or both.

(2) Separational mechanisms which lead to an annular layer of the low-viscosity solvent may be present. Most of the shearing action could take place in this layer, thus developing an effective 'slip' mechanism at the tube wall.

(3) The continuum (viscoelastic) properties of the fluid may account for the increased stability of the laminar flow field and the low pressure losses under turbulent conditions.

These possible causes will be discussed in turn.

Particulate effects

The effects of the addition of small particles to a fluid stream have been studied by a number of investigators. Vanoni (1946) and Sproull (1961) report modest—7–22 %—reductions in turbulent drag upon addition of solids.† Others (Elata & Ippen 1961; Daily & Chu 1961; Ismail 1952; Kada & Hanratty 1960) report drag coefficients which are generally either identical with those of Newtonian fluids or slightly higher. These latter investigators dealt, in part, with more concentrated systems for which unavailable rheological measurements may be necessary to enable a proper correlation of the data to be made, but in any event do not depict results comparable to the data shown in figure 1. Their observations of increases in the turbulent intensity, at least within the large-scale end of the spectrum, are also not supported by preliminary measurements in viscoelastic polymeric

† Sproull also reports 'viscosity decreases' of a larger magnitude under laminar flow conditions. The experimental technique used to study the rapidly settling suspensions raises a number of significant questions, however, and the results are so contrary to all other knowledge of the behaviour of dilute laminar suspensions that it would appear possible to discount Sproull's laminar measurements completely. (Elata & Ippen (1961) and Thomas (1963) give good discussions of the viscous behaviour of dilute suspensions.)

systems, although the data are perhaps inconclusive (Anderson & Rau 1956; Shaver & Merrill 1959). In all cases the differences between the behaviour of turbulent suspensions and Newtonian fluids were found to decrease with increasing Reynolds number. In concentrated suspensions, on the other hand, the numerous available studies (Dodge 1957; Thomas 1962, 1963; Bogue & Metzner 1963; Eissenberg & Bogue 1963) show the turbulent drag coefficients to be identical with those of purely viscous fluids as indicated in figure 1. Thus, none of the available experimental studies in which particulate effects may be of importance lead to results comparable to those depicted by the data reported in figure 1.

Saffman (1962) has published an analysis which indicates that stabilization of laminar flow and dampening of turbulence may occur in suspensions but only if the density of the particles, or the time scale of their fluctuations, is large as compared with that of the turbulent flow field. This does not appear to be the situation which occurs in polymeric dispersions or solutions.

It should be observed that the effects reported in figure 1 are due only to certain polymeric additives. Other polymeric materials studied by Savins (1963) and by Dodge & Metzner (1959) show only the effects common to purely-viscous fluids. Correspondingly, the degree to which polymeric additives reduce turbulent drag varies greatly with the kind of molecule used (Dodge 1957; Sailor 1960; Crawford 1962; Savins 1963; Park 1963). Thus, if particulate effects are to account for this behaviour they must be centred in effects considerably more sophisticated than molecular size and density, and may involve factors such as molecular configuration and flexibility.

Effects somewhat similar to those depicted in figure 1 have recently been reported by Bobkowicz & Gauvin (1963) using small and highly asymmetric nylon fibre suspensions; while the observed drag reductions were not nearly as large as those noted to date in polymeric solutions it is not clear that the basic causes are different. Whether these fibres and molecules act as discrete particles, or whether the change in drag characteristics is reflected in the continuum properties of such solutions or suspensions, is not known at present. Thus, further consideration of both particulate and continuum properties would appear to be desirable; the present paper takes one step in the direction of the latter.

Slip mechanisms, wall effects

The movement of macromolecular or particulate matter away from the wall of a tube, due to gross entanglement of fibres or particles with one another or due to hydrodynamic forces on the molecules or particles, enables the generation of a low-viscosity layer in which a major portion of the shearing takes place. (Paper pulps flowing in open channels reveal such separation effects strikingly.) As a low-viscosity annular layer of small width may remain laminar at high flow rates of the bulk of the material through the duct, the pressure drop of suspended materials 'slipping' through the tube by means of this mechanism may be very low. The subject has been extensively studied (Mooney 1931; Schultz-Grunow 1958; see also Oldroyd 1956; Metzner 1961) and the underlying mechanisms are well understood. The effect is manifested macroscopically as a pronounced 'diameter effect' under laminar flow conditions. Reference to figure 1, as well as to more extensive laminar flow measurements on similar systems (Dodge 1957; Sailor 1960; Savins 1963), shows that this is clearly not the situation of interest herein. Furthermore, turbulence in either the annular layer or the core or bulk of the fluid would serve to mix the core with any such annular layer and progressively reduce its effectiveness at increasing Reynolds numbers. For both of these reasons it is thus seen that such a mechanism cannot be the cause of the present phenomenon.

Continuum properties

While the outstandingly viscoelastic nature of solutions which exhibit the low drag coefficients f portrayed in figure 1 suggests that continuum viscoelastic properties may be responsible for the observed phenomenon, in order to test such a hypothesis the drag reduction must be characterized quantitatively. This may be done by comparing the actual drag coefficient with the values which would be obtained in the limiting cases of zero and perfect turbulence suppression. The former value may be obtained from the curves for purely viscous fluids of identical flow behaviour index; the latter from an extension of the laminar line

$$(f_l = 16/N'_{Re}).$$

Thus the ratio $(f_{pv}-f)/(f_{pv}-f_l)$ may be used to express the fractional reduction in the drag coefficient. Here the drag coefficient f is defined as

$$(D\Delta P/4L)/(\rho V^2/2g_c),$$

where ΔP is the pressure drop in a tube of length L; the subscripts pv and l refer to purely-viscous fluids and laminar flow conditions, respectively.

The factors used to define, quantitatively, the viscoelasticity of the fluid will depend upon the particular constitutive equation chosen to represent the behaviour of the material. As discussed elsewhere (White & Metzner 1963; Ginn & Metzner 1963; Tanner 1963), no clear choice is as yet possible, but the equation

$$\tau^{ij} = 2\mu d^{ij} - \frac{\mu}{G}\frac{\delta\tau^{ij}}{\delta t}, \tag{2}$$

in which $\delta/\delta t$ denotes the usual convected derivative (Oldroyd 1950), τ^{ij} is the stress tensor and d^{ij} the rate of strain tensor, appears to hold promise at least for purposes of approximation. Here μ denotes the variable viscosity (dependent on the invariants of the stress or strain-rate matrices) and G denotes the elastic modulus of the material. Under conditions of steady laminar shearing motion this equation yields, using the usual Cartesian co-ordinate system with Γ as the rate of shear,

$$\tau_{12} = \mu\Gamma, \tag{3a}$$

$$P_{11} - P_{22} = 2\tau_{12}^2/G, \tag{3b}$$

$$P_{22} = P_{33}, \tag{3c}$$

and zero values for all other components of the deviatoric stress tensor P_{ij}.† Thus, these equations provide a means for evaluating, from well-defined experi-

† For purposes of comparison it may be noted that in such steady-flow fields the deviatoric normal stress terms P_{ii} would be identically equal to zero for Newtonian fluids as well as for purely-viscous non-Newtonians as defined herein.

ments carried out under laminar flow conditions, the parameters μ and G. From (2) one sees that these same parameters serve to define completely the fluid properties under all flow conditions, including the unsteady conditions of interest in the turbulent régime.

The influence of the viscosity function μ may be considered, as in the case of purely-viscous fluids, through use of the generalized Reynolds number N'_{Re} and the flow behaviour index n'. The additional influence of fluid elasticity may be incorporated by use of any one of the three possible dimensionless ratios involving the viscoelastic parameter or stresses, namely $G/\rho V^2$, G/τ_{12} or $(P_{11}-P_{22})/\tau_{12}$. The last of these groups, representing the ratio of elastic to viscous stresses developed by the fluid under conditions of steady laminar flow, perhaps depicts the comparative importance of elastic and viscous stresses most clearly, ranging as it does from a value of zero for purely viscous systems toward infinity as fluid elasticity increases. Additionally it represents the physical-property ratio which is most directly determined experimentally. For these reasons it was chosen in the present analysis to depict the comparative importance of elastic and viscous forces at any given stress level τ_{12}.

If one thus assumes that viscoelasticity is the sole cause of the turbulent drag reduction being considered in the present paper, and that the above equations serve to define the fluid physical properties completely, one may write†

$$(f_{pv}-f)/(f_{pv}-f_l) = \phi(N'_{Re}, n', (P_{11}-P_{22})/\tau_{12}), \tag{4}$$

where ϕ is an unspecified function. Correspondingly, if a more complex constitutive equation is chosen to represent the fluid properties additional dimensionless groups, embodying ratios of the additional physical parameters, will be required. In this sense (4) may be considered to represent a first approximation to a complete or final equation. Methods for obtaining the coefficients in more complex equations are discussed elsewhere (Ginn & Metzner 1963; Markovitz 1962; Metzner 1961).

Since the ratio $(P_{11}-P_{22})/\tau_{12}$ varies with shear stress or shear rate and since $(P_{11}-P_{22})$ approaches zero more rapidly than does τ_{12} at low shear stresses or shear rates (Markovitz & Brown 1963; Ginn & Metzner 1963; Metzner, Houghton, Sailor & White 1961) this ratio will increase from small values near the centreline of the tube and reach a maximum value at the tube wall. The most sensitive or most correct radial position at which to evaluate this ratio, if there is one, is unknown; hence it will arbitrarily be evaluated at the tube wall in the present analysis.

At given values of the Reynolds number and flow index, the drag-coefficient ratio in (4) will obviously be a function of this last dimensionless group $(P_{11}-P_{22})/\tau_{12}$. Increasing the tube diameter, under these conditions, decreases this ratio of elastic to viscous stresses, since the shearing stress decreases with increasing tube diameter at a given Reynolds number. Therefore, if increasing values of this ratio are to portray increased viscoelastic drag reduction characteristics, the drag

† No essential features are lost if the equation is written down intuitively. Obviously the same or an equivalent result may be obtained by employing the equations of motion or the Π theorem.

reduction ratio should increase with decreasing tube diameter (or, alternately, the drag coefficient f should decrease with decreasing tube diameter) at a given Reynolds number. The data of figure 1, as well as those of Savins (1963), Dodge (1957) and Sailor (1960) are in qualitative agreement with such a trend.

Finally, it should be noted that if continuum properties of the fluid are responsible for the observed turbulent behaviour of polymeric solutions, it is not necessary to inquire about the relationship of molecular parameters (size, shape, flexibility, etc.) to the turbulent processes: equations such as (4) arising from a proper combination of the equations of motion and the chosen constitutive equation will suffice to interpret the turbulent data. Measurement of the parameters in the constitutive equation will, of course, be necessary in any event. The relationship between these constitutive parameters and molecular structure may represent a worthwhile area of investigation for other purposes, such as synthesis of new and more effective polymeric molecules, but it should be emphasized that this represents a problem which is quite distinct from that of interpreting and understanding the turbulent behaviour of any continuum.

Experimental

Measurement of the shearing stress τ_{12} as a function of the invariants of the strain-rate tensor, or, equivalently, as a function of shear rate in a simple laminar shearing flow experiment, poses only few problems in either principle or practice (Metzner 1961). The same is not true of the normal stress difference, as the usual rheological techniques are restricted to shear rates well below those of interest in most fluid-mechanics problems (White & Metzner 1962). Birefringence techniques extend into the range of shear rates of interest, but have not been extended or developed to cover it all; hence the recently-developed 'jet-thrust' device (Shertzer & Metzner 1963; see also Metzner *et al.* 1961) was employed. The principle involved is that of a direct measurement of the axial normal stress and momentum flux of a laminar jet of fluid issuing from a smooth, round tube. As the details of this technique have been published recently, it will not be described in detail.

The drag coefficients were measured using the routine procedures and equipment described previously (Dodge & Metzner 1959) and these also need not be discussed except to point out that careful calibrations of the magnetic flowmeter were made and the experimental techniques were checked by first obtaining data on both Newtonian and non-Newtonian purely-viscous fluids. Full details are available (Park 1963).

Experimental results

Table 1 compares the results obtained with a variety of polymeric solutions. In order to make a detailed comparison with (4) possible not only the Reynolds number but also the flow index should be either maintained constant or varied sufficiently to ascertain its influence. The data in table 1 are not sufficiently extensive to achieve such a result, yet the trend toward increased drag reduction with increased values of the ratio of elastic to normal stresses appears to be clearly defined.

It is interesting to view these results in the light of the suggestion that molecular entanglement, as measured by intrinsic viscosity, may represent a guide to the drag-reduction characteristics of polymer molecules (Fabula 1963). The intrinsic viscosity of the solutions represented in table 1 were not measured precisely but the values given in table 2 are sufficiently accurate to enable a test of this conjecture. It is seen that the Carbopol, which exhibits little or no drag-reduction at the concentration levels used,† forms notoriously viscous solutions at low concentration levels. The most effective additive, J-100, has a comparable intrinsic viscosity, while Polyox, another good drag-reducing agent, exhibits the lowest intrinsic viscosity. Data on another series of additives (Crawford 1962) similarily appear to show no relation between viscosity and drag-reduction. Thus, while the results available may not be sufficiently precise to reach unequivocal conclusions, there seems to be no general support for the suggestion that intrinsic viscosity is a relevant parameter. This is perhaps not surprising since viscosity (i.e. shear-stress–shear-rate) data are generally only insensitive tests of fluid properties or of constitutive equations.

Fluid	n'	$(P_{11}-P_{22})/\tau_{12}$	$(f_{pv}-f)/(f_{pv}-f_l)$
0·3 % J-100	0·55	29·0	0·88
1·9 % Polyox†	0·52	8·2	0·58
1·5 % Polyox†	0·70	5·0	0·56
1·18 % Polyox†	0·63	7·4	0·34
0·45 % CMC†	0·63	0·25	0·27
0·60 % Carbopol	0·60–0·90	0 (±0·10)	0

† Data of Sailor (1960).

TABLE 1. Viscoelastic nature of friction-reducing agents

All data correspond to $N'_{Re} = 10^4$.

Fluid	Relative intrinsic viscosity
J-100	1·0
Polyox	0·1
CMC	0·2
Carbopol	1·0

TABLE 2

Figure 2 depicts the normal-stress and shear-stress (rheological) measurements for the same J-100 solution used to obtain the turbulent data of figure 1. The curves shown were computer-fitted using a regression analysis. The highest shear-stress points, for any given tube diameter, were subject to some correction due to effects at the tube inlet. As these inlet effects are not well-defined for viscoelastic fluids, accurate corrections could not be made to such data. As a result these points were simply weighted less heavily in the regression analysis than the remaining points for which end-effect corrections are properly negligible (Park 1963).

† That is, it approximates the f–N'_{Re}–n' characteristics of purely viscous fluids such as slurries.

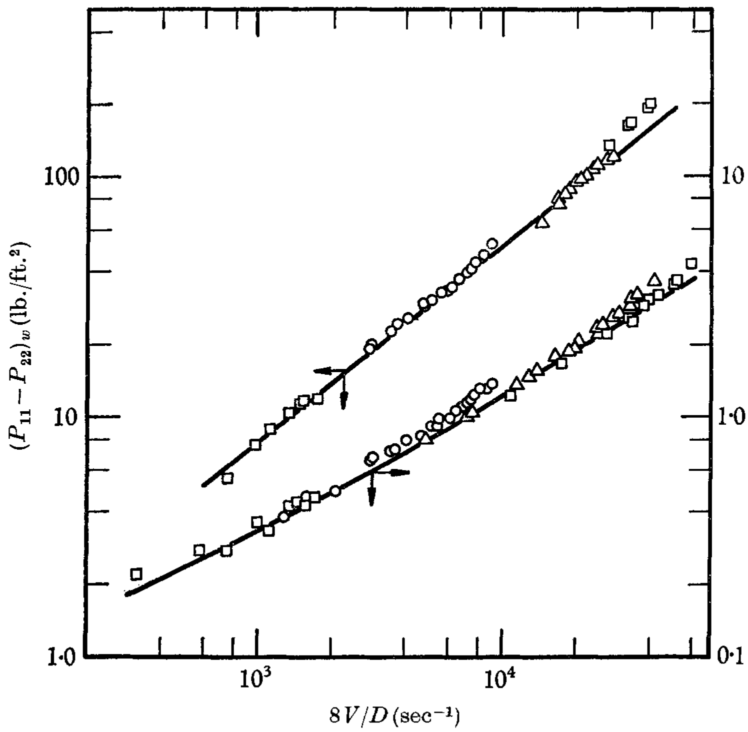

FIGURE 2. Rheological properties of fluid used to obtain data of figure 1.

	Tube diameter used (mm)
□	5·422
○	2·668
△	1·146
◻	0·8350

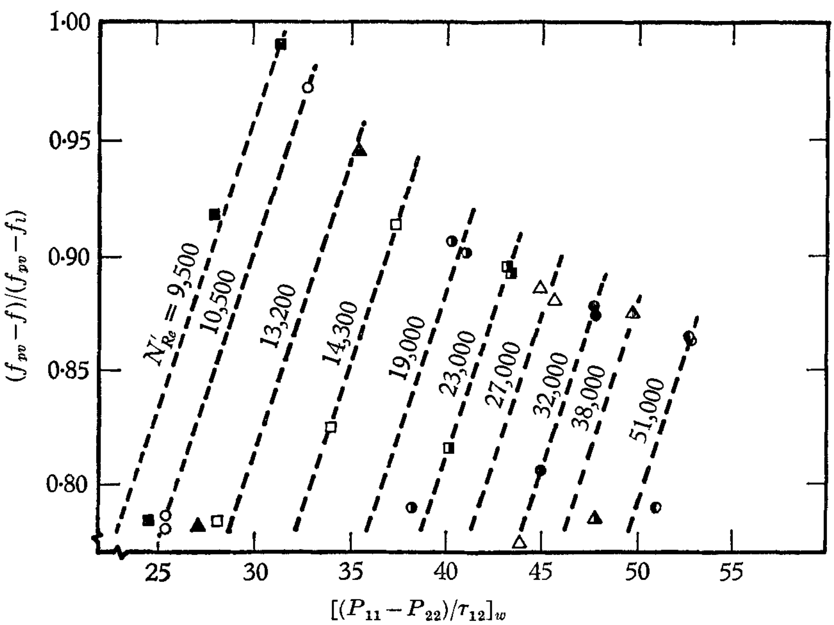

FIGURE 3. Dependence of drag-reduction effects on ratio of elastic to viscous forces.

Since the curvature of the shear-stress–shear-rate curve of figure 2 is not very great, the flow index is approximately constant at a given Reynolds number. Under these conditions, equation (4) indicates that the drag reduction ratio should be a unique function of the ratio of elastic to viscous stresses, with Reynolds number as a parameter. Figure 3 depicts the results of figures 1 and 2

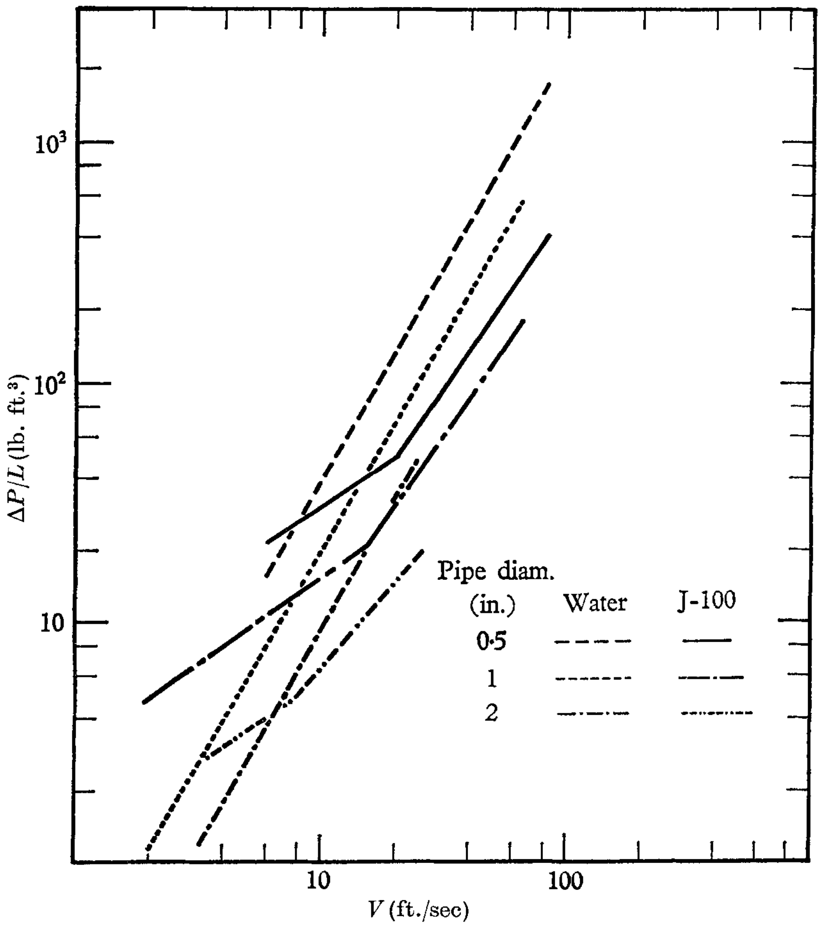

FIGURE 4. Pressure drop *vs* flow-rate (from figure 1) illustrating actual magnitudes of the decreased drag of viscoelastic fluids.

in this form. Since only one fluid was studied it cannot be claimed that the trends shown represent any unique correlation, since all terms, other than the diameter, which appear in the Reynolds number and in the ratio $(P_{11}-P_{22})/\tau_{12}$ were not varied separately. Obviously additional data, using other fluids, are needed.

Figure 4 compares the actual pressure-drop measurements with those of water. It is seen that appreciable reductions in pressure drop, under turbulent conditions, accompany the addition of the polymer. While the actual reductions in pressure drop are large, it is to be noted that others (Lummus *et al.* 1961; Crawford 1962; Fabula 1963; Savins 1963) have observed reductions which were, occasionally, as great or even greater when only very small concentrations of polymer (as little as a few parts per million) were used. The reasons for the comparatively small improvement in the drag reduction with increasing concentration lies in the fact that two opposing factors are operative: increasing the polymer concentration increases the elastic forces but the viscous forces increase simultaneously, and, above some concentration level characteristic of the system used, these increases in viscosity may overshadow the effects of higher fluid elasticity. In the present study an abnormally high concentration of polymer was used to aid the problem of determination of normal stresses: presently-available techniques do not suffice to obtain normal stress data such as those of figure 2 on very dilute solutions for which the magnitudes of the stresses (although not necessarily their ratio) are greatly reduced. Thus, improved rheological instrumentation remains as a continued need of further studies in this area.

In conclusion, we may say that a test has been made of the hypothesis that the turbulent-drag-reduction characteristics of polymeric additives are a result of their viscoelastic properties. The test shows that the available data are in agreement with such a hypothesis but further studies are required for conclusive results.

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The results presented would have been unattainable several years ago in the absence of a technique for the measurement of normal stresses at the high shear-rates of interest in this work. Accordingly, the contributions of our several co-workers concerned with this problem are especially acknowledged.

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